RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FOURTH SEMESTER EXAMINATION, May 2019

SECOND YEAR [BATCH 2017-19]

Date : 10/05/2019 **MATHEMATICS**

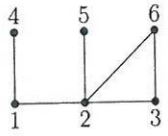
Time : 11 am – 1 pm Paper : XVII [GRAPH THEORY & COMBINATORICS] Full Marks : 50

Answer **any ten** questions : [10×5]

1. a) Define a complete graph. Find the number of edges in a complete graph with 2000 vertices. [1+1]

   b) Is the sequence 4,3,3,3,2,2,2,1 graphical? Justify your answer. [3]

2. a) List the eccentricities of all the vertices of the graph drawn below. [2]

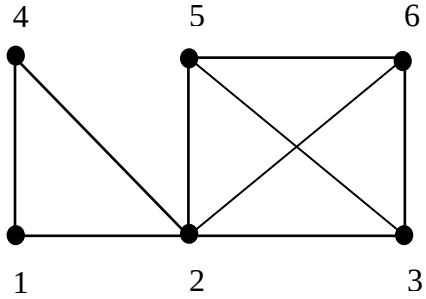


   b) Find the center of the graph given above. [2]

   c) What are the radius and diameter of the graph given above? [1]

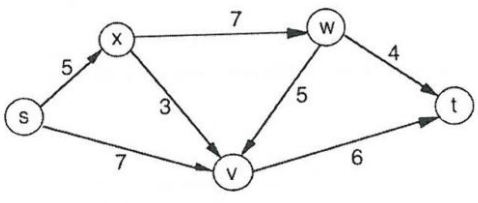
3. a) Distance matrix of a graph G with n vertices is $\mathcal{D}(G)=\left(d_{i,j}\right)_{n\times n}$, where $d_{i,j}$ is the distance between two vertices i and j. Construct distance matrix of the following graph. [3]

G :



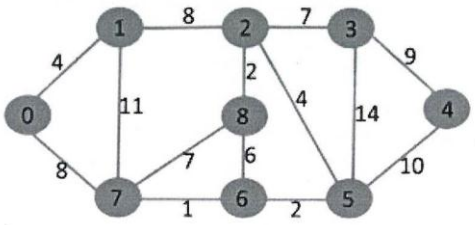
   b) Define a bipartite graph.Count the number of cycles of length 63 in the complete bipartite graph $K_{87,121}$. [1+1]

4. Consider the flow network depicted in the figure below. Using Ford-Fulkerson algorithm find the optimal flow in the network. [5]



5. The cycle graph of order 5, C_5 is a 3-chromatic graph. Using Mycielski's construction produce a 4 chromatic graph from C_5. [5]

6.



Consider the above graph. Applying Kruskal's theorem generate the minimum spanning tree of the graph. [5]

7. Let $S=\{0,1,2,3,4,5\}$ and a function $f:S\to S$ be defined by $f(x)=x+3(\mathrm{mod}(5))$.

The functional digraph of a function f is a digraph consists of the vertex set S and the edge set

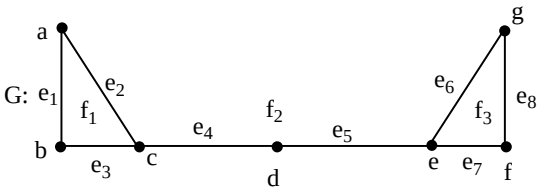
$\{(x,f(x)):x\in S\}$.

a) Write down the adjacency matrix of the functional digraph of f. [2]

b) List the in-degrees and out-degrees of the vertices. [2]

c) Is the digraph Eularian? [1]

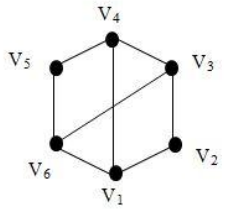
8. a) Draw the dual graph of the following graph. [2]



b) Define a k-critical graph. If G is a k-critical graph, then prove that $\delta(G)\geq k-1$, where $\delta(G)=\min\{d(v):v\in V(G)\}$. [1+2]

9. a) Find the complement of the graph [2]

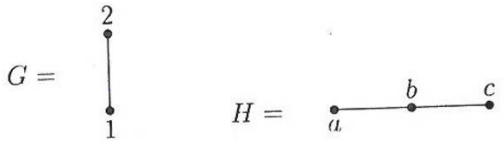
G:



b) Prove that a tree with n-vertices has (n-1) edges. [3]

10. Define independence number of a graph. If G is a simple graph, then prove that $\chi(G)\geq\frac{n(G)}{\alpha(G)}$, where n(G) is the number of vertices of G, $\alpha(G)$ is the independence number of G and $\chi(G)$ is the chromatic number of G. [1+4]

11. Graph, G and H are drawn below



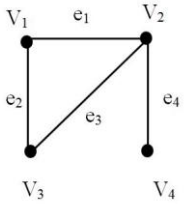
a) Draw the graph of $G \square H$. [3]

b) Find $\chi(G \square H)$. [2]

12. Define planar graph. Show that K_5 is non-planar. [1+4]

13. a) Find the incidence matrix of the graph [2]

G:



b) Draw the digraph of which $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 0 & 0 \end{pmatrix}$ is the adjacency matrix. [3]

14. a) Define Stirling numbers of first kind. Show that the Stirling numbers of first kind satisfy the recurrence relation $s_{n+1}^k = s_n^{k-1} - n s_n^k$, $1 \le k \le n$ with $s_{n+1}^0 = 0, s_{n+1}^{n+1} = 1$ [1+2]

b) The Stirling numbers of first kind for $n = 4$ is given in the table below.

k =	0	1	2	3	4
n = 4	0	6	11	6	1

Write down the Stirling numbers of first kind for $n = 5$. [2]

15. Let S_n^k denote the number of partitions of a set with n-elements into k-parts. Prove that $S_n^1 = 1, S_n^n = 1$ and $S_n^k = S_{n-1}^{k-1} + k S_{n-1}^k$, $2 \le k \le n-1$. [5]

16. a) Let G be a connected graph with at least two vertices. If the number of edges in G is less than the number of vertices of G, then show that G has a pendant vertex. [3]

b) How many spanning tree does $K_{2,3}$ have? [2]

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